Comparison of Conjugate Gradient Method for Nonsymmetric matrices

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In this talk, we show some comparisons of Conjugate Gradient (CG) methods for nonsymmetric matrices. Even though the CG method is for symmetric matrices, it can be applied to nonsymmetric matrices with some modifications. Since there are few reports on the comparison of their convergence behavior on the modern computers, we tested the following two methods:

- CGNR: Conjugate Gradient method for the normal equation $A'Ax = A'b$[2]
- Conjugate Gradient method to solve $Ax = b$ and $A'y = c$ simultaneously[1]

The condition number of $A'A$ is a square of that of original matrix A, which may lead to poor convergence for the CGNR method. However, there are some test problems where it shows smoother and even faster convergence behavior than the BiCG method to $Ax = b$. Second, to solve $Ax = b$ and $A'y = c$ simultaneously, the dimension of a coefficient matrix becomes twice of that of original matrix A and A'. In this case, there is a difficulty to give a right-hand side of additional equation $A'y = c$, but the convergence is not as bad as expected.
We will also show results of preconditioned methods and ones with the use of quadruple precision operations for accurate computation. To perform these numerical experiments effectively, we used a tool called SILC[3].

References

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